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MATHEMATICS

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AIM POINTS
MATHEMATICS
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XIth, XIIth, TARGET IIT-JEE (MAIN + ADVANCE) & COMPETITIVE EXAM. FOR XII (PQRS)

INDEFINITE INTEGRALS

& Their Properties

CONTENTS

THINGS TO REMEMBER

1. $\frac{d}{dx}\left(\frac{x^{n+1}}{n+1}\right) = x^n,\ n \neq 1$ $\Rightarrow$ $\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$

2. $\frac{d}{dx}(\log x) = \frac{1}{x}$ $\Rightarrow$ $\int \frac{1}{x} dx = \log|x| + c$

3. $\frac{d}{dx}(e^x) = e^x$ $\Rightarrow$ $\int e^x\, dx = e^x + C$

4. $\frac{d}{dx}\left(\frac{a^x}{\log_e a}\right) = a^x, a > 0, a \neq 1$ $\Rightarrow$ $\int a^x\, dx = \frac{a^x}{\log a} + C$

5. $\frac{d}{dx}(-\cos x) = \sin x$ $\Rightarrow$ $\int \sin x\, dx = -\cos x + C$

6. $\frac{d}{dx}(\sin x) = \cos x$ $\Rightarrow$ $\int \cos x\, dx = \sin x + C$

7. $\frac{d}{dx}(\tan x) = \sec^2 x$ $\Rightarrow$ $\int \sec^2 x\, dx = \tan x + C$

8. $\frac{d}{dx}(-\cot x) = \text{cosec}^2 x$ $\Rightarrow$ $\int \text{cosec}^2 x\, dx = -\cot x + C$

9. $\frac{d}{dx}(\sec x) = \sec x \tan x$ $\Rightarrow$ $\int \sec x \tan x\, dx = \sec x + C$

10. $\frac{d}{dx}(-\text{cosec } x) = \text{cosec } x \cot x$ $\Rightarrow$ $\int \text{cosec } x \cot x\, dx = -\text{cosec } x + C$

11. $\frac{d}{dx}(\log \sin x) = \cot x$ $\Rightarrow$ $\int \cot x\, dx = \log|\sin x| + C$

12. $\frac{d}{dx}(-\log \cos x) = \sin x$ $\Rightarrow$ $\int \tan x\, dx = -\log|\cos x| + C$

13. $\frac{d}{dx}(\log(\sec x + \tan x)) = \sec x$ $\Rightarrow$ $\int \sec x\, dx = \log|\sec x + \tan x| + C$

14. $\frac{d}{dx}(\log(\text{cosec } x - \cot x)) = \text{cosec } x$ $\Rightarrow$ $\int \text{cosec } x\, dx = \log|\text{cosec } x - \cot x| + C$

15. $\frac{d}{dx}\left(\sin^{-1}\frac{x}{a}\right) = \frac{1}{\sqrt{a^2 - x^2}}$ $\Rightarrow$ $\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + C$

16. $\frac{d}{dx}\left(\cos^{-1}\frac{x}{a}\right)=-\frac{1}{\sqrt{a^2-x^2}}$ $\Rightarrow \int -\frac{1}{\sqrt{a^2-x^2}}dx=\cos^{-1}\left(\frac{x}{a}\right)+C$

17. $\frac{d}{dx}\left(\frac{1}{a}\tan^{-1}\frac{x}{a}\right)=\frac{1}{a^2+x^2}$ $\Rightarrow \int \frac{1}{a^2+x^2}dx=\tan^{-1}\left(\frac{x}{a}\right)+C$

18. $\frac{d}{dx}\left(\frac{1}{a}\cot^{-1}\frac{x}{a}\right)=-\frac{1}{a^2+x^2}$ $\Rightarrow \int -\frac{1}{a^2+x^2}dx=\cot^{-1}\left(\frac{x}{a}\right)+C$

19. $\frac{d}{dx}\left(\frac{1}{a}\sec^{-1}\frac{x}{a}\right)=\frac{1}{x\sqrt{a^2+x^2}}$ $\Rightarrow \int \frac{1}{x\sqrt{a^2+x^2}}dx=\sec^{-1}\left(\frac{x}{a}\right)+C$

20. $\frac{d}{dx}\left(\frac{1}{a}\operatorname{cosec}^{-1}\frac{x}{a}\right)=-\frac{1}{x\sqrt{a^2+x^2}}$ $\Rightarrow \int -\frac{1}{x\sqrt{a^2+x^2}}dx=\operatorname{cosec}^{-1}\left(\frac{x}{a}\right)+C$

21. $\frac{d}{dx}\{\phi(x)+C\}=\phi'(x)=f(x)$

22. $\int_a^b f(x)dx=\phi(x)+C$

Following are indefinite integrals of some standard functions :

23. $\int x^n\,dx=\frac{x^{n+1}}{n+1}+C, n\neq -1$

24. $\int \frac{1}{x}dx=\log_e x+C$

25. $\int a^x\,dx=\frac{a^x}{\log_e a}+C, a\neq 1, a>1$

26. $\int e^x\,dx=e^x+C$

27. $\int \sin x\,dx=-\cos x+C$

28. $\int \cos x\,dx=\sin x+C$

29. $\int \sec^2 x\,dx=\tan x+C$

30. $\int \operatorname{cosec}^2 x\,dx=-\cot x+C$

31. $\int \sec x\tan x\,dx=\sec x+C$

32. $\int \operatorname{cosec} x\cot x\,dx=-\operatorname{cosec} x+C$

33. $\int \tan x\,dx=\log|\sec x|+C$

34. $\int \cot x \, dx = \log | \sin x | + C$

35. $\int \sec x \, dx = \log | \sec x + \tan x | + C = \log \left| \tan\left(\frac{\pi}{4} + \frac{x}{2}\right) \right| + C$

36. $\int \text{cosec} \, dx = \log | \text{cosec} - \cot x | + X = \log \left| \tan \frac{x}{2} \right| + C$

37. $\int (ax + b)^n \, dx = \frac{1}{a} \frac{(ax+b)^{n+1}}{n+1} + C$

38. $\int \frac{1}{ax+b} dx = \frac{1}{a} \log | ax + b | + C$

39. $\int a^{bx+c} \, dx = \frac{1}{b} \cdot \frac{a^{bx+c}}{\log_e a} + c, b > 0, b \neq 1$

40. $\int e^{bx+c} \, dx = \frac{1}{b} e^{bx+c} + C$

41. $\int \sin(ax + b) \, dx = -\frac{1}{a} \cos(ax + b) + C$

42. $\int \cos (zx + b) \, dx = \frac{1}{a} \sin(ax + b) + C$

43. $\int \tan(ax + b) \, dx = \frac{1}{a} \log | \sec(ax + b) | + C$

44. $\int \cot(ax + b) \, dx = \frac{1}{a} \log | \sin(ax + b) | + C$

45. $\int \sec^2(ax + b) = \frac{1}{a} \tan(ax + b) + C$

46. $\int \text{cosec}^2(ax + b) = -\frac{1}{a} \cot(ax + b) + C$

47. $\int \sec(ax + b) \tan(ax + b) \, dx = \frac{1}{a} \sec(ax + b) + C$

48. $\int \text{cosec}(ax + b) \tan(ax + b) \, dx = -\frac{1}{a} \text{cosec}(ax + b) + C$

49. $\int \sec(ax + b) \, dx = \frac{1}{a} \log | \sec(ax + b) + \tan(ax + b) + C$

50. $\int \text{cosec}(ax + b) \, dx = \frac{1}{a} \log | \text{cosec}(ax + b) + \cot(ax + b) + C$

51. $\int \frac{1}{a^2+x^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a}+C$

52. $\int \frac{1}{x^2-a^2}dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right|+C$

53. $\int \frac{1}{x^2-a^2}dx = \frac{1}{2a}\log\left|\frac{x+a}{x-a}\right|+C$

54. $\int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\frac{x}{a}+C$

55. $\int \frac{1}{\sqrt{a^2+x^2}}dx = \log\left|x+\sqrt{a^2+x^2}\right|+C$

56. $\int \frac{1}{\sqrt{a^2-x^2}}dx = \frac{x}{2}\sqrt{a^2-x^2}+\frac{1}{2}a^2\sin^{-1}\frac{x}{a}+C$

57. $\int \sqrt{a^2-x^2}dx = \frac{1}{2}x\sqrt{a^2-x^2}-\frac{1}{2}a^2\log\left|x+\sqrt{a^2-x^2}\right|+C$

58. $\int \sqrt{a^2+x^2}dx = \frac{1}{2}x\sqrt{a^2+x^2}-\frac{1}{2}a^2\log\left|x+\sqrt{a^2+x^2}\right|+C$

59. (i) $\int e^x \{f(x) + f'f(x)\}\, dx = e^x f(x) + C$

(ii) $\int e^{kx} \{f(x) + f'f(x)\}\, dx = e^{kx} f(x) + C$

60. (i) $\int e^{ax} \sin(bx + c)\, dx = \frac{e^{ax}}{a^2+b^2}\{a \sin(bx + c) - b \cos(bx + c)\} + \lambda$

(ii) $\int e^{ax} \cos(bx + c)\, dx = \frac{e^{ax}}{a^2+b^2}\{a \cox(bx + c) + b \cos(bx + c)\} + \lambda$

61. In order to evaluate integrals of the form $\int \frac{\phi(x)}{P\sqrt{Q}}dx$, where

(i) P and Q both are linear functions of x, we put $Q = T^2$

(ii) P is quadratic and Q is a linear function of x, we put $Q = t^2$

(iii) P is linear and Q is quadratic function of x, we put $P = \frac{1}{t}$

(iv) P and Q both are pure quadratic of the form $P = ax^2 + b$ and $Q = cx^2 + d$, then we substitute $x = \frac{1}{t}$ and $c = dt^2 = u^2$.

EXERCISE-1

1. Evaluate the following integrals :

(i) $\int x^4\,dx$ (ii) $\int \sqrt{x}\,dx$ (iii) $\int \frac{1}{\sqrt{x}}dx$ (iv) $\frac{1}{x^3}dx$

(v) $\int a^{3\log_e x}dx$

2. Evaluate : $\int \frac{e^{5\log_e x}-e^{4\log_e x}}{e^{3\log_e x}-e^{2\log_e x}}dx$

3. Evaluate : $\int \frac{2}{1-\cos 2x}dx$

4. Evaluate :

(i) $\int \frac{\cos 2x+2\sin^2 x}{\cos^2 x}$

(ii) $\int \frac{2\cos^2 x-\cos 2x}{\sin^2 x}$

5. Evaluate : $\int \frac{e^{6\log_e x}-e^{5\log_e x}}{e^{4\log_e x}-e^{3\log_e x}}dx$

6. Evaluate : $\int \frac{\cos 2x+2\sin^2 x}{\sin^2 x}dx$

7. Evaluate :

(i) $\int 3\sin x-2\cos x+4\sec^2 x-5\,\mathrm{cosec}^2 x\,dx$

(ii) $\int \sqrt{1+\cos 2x}\,dx$

(iii) $\int \sqrt{1-\sin 2x}\,dx$

(iv) $\int \frac{\cos x-\cos 2x}{1-\cos x}dx$

8. Evaluate :

(i) $\int \tan^2 x\,dx$ (ii) $\int \cot^2 x\,dx$

(iii) $\int \frac{1}{\sin^2 x\cos^2 x}dx$ (iv) $\int \frac{\cos 2x}{\sin^2 x\cos^2 x}dx$

(v) $\int \frac{2+3\cos x}{\sin^2 x}dx$

(vi) $\int (2\tan x - 3\cot x)^2\,dx$

9. Evaluate :

(i) $\int \frac{1}{1+\sin x}dx$

(ii) $\int \frac{1}{1+\cos x}dx$

10. Evaluate :

(i) $\int \frac{\sin^6 x+\cos^6 x}{\sin^2 x\cos^2 x}dx$

(ii) $\int \frac{1+\cos 4x}{\cot x-\tan x}dx$

11. Evaluate :

(i) $\int \sin^{-1}(\cos x)\,dx,\ 0 \le x \le \pi$

(ii) $\int \tan^{-1}\left\{\sqrt{\left(\frac{1-\cos 2x}{1+\cos 2x}\right)}\right\}dx, 0 < x < \frac{\pi}{2}$

(iii) $\int \tan^{-1}(\sec x + \tan x)\,dx, -\frac{\pi}{2} < x < \frac{\pi}{2}$

12. Evaluate : $\int \tan^{-1}\left\{\sqrt{\frac{1-\sin x}{1+\sin x}}\right\}dx, -\frac{\pi}{2} < x < \frac{\pi}{2}$.

13. Evaluate :

(i) $\int e^{x\log a} + e^{a\log x} + e^{a\log a}\,dx$

(ii) $\int \left(\frac{x}{m}+\frac{m}{x}+x^m+m^x\right)dx$

14. Evaluate : $\int \frac{2^x+3^x}{5^x}dx$

15. Evaluate : $\int \left\{\sqrt{x}(ax^2+bx+c)\right\}dx$

16. Evaluate : $\int \left(\frac{x}{m}+\frac{m}{x}+x^m+mx\right)dx$

17. Evaluate : $\int \left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2 dx$

18. Evaluate : $\int (x^e + e^x + e^e)\,dx$

19. Evaluate : $\int \frac{x^6+1}{x^2+1}dx$

20. Evaluate : $\int \frac{x^{1/3}+\sqrt{x}+2}{\sqrt[3]{x}}dx$

21. Evaluate : $\int \sqrt{x}\,(3-5x)\,dx$

22. Evaluate : $\int (3x+4)^2\,dx$

23. Evaluate : $\int \frac{\sin^3 x-\cos^3 x}{\sin^2 x\cos^2 x}dx$

24. Evaluate : $\int \frac{5\cos^6 x+6\sin^3 x}{2\sin^2 x\cos^2 x}dx$

25. Evaluate : $\int \frac{1}{1-\cos 2x}dx$

26. Evaluate : $\int \frac{\cos x}{1+\cos x}dx$

27. Evaluate : $\int \frac{1-\cos x}{1+\cos x}dx$

28. Evaluate : $\int \frac{\cos 2x+2\sin^2 x}{\cos^2 x}dx$

29. Write the primitive or anti–derivative of $f(x)=\sqrt{x}+\frac{1}{\sqrt{x}}$.

30. Evaluate : $\int e^{2x-3}\,dx$

31. Evaluate : $\int \sin(ax+b)\cos(ax+b)\,dx$

32. Evaluate : $\int \frac{\sin^8 x-\cos^8 x}{1-2\sin^2 x\cos^2 x}dx$

33. Evaluate : $\int \frac{1}{\sqrt{3x+4}-\sqrt{2x+1}}=dx$

34. Evaluate :

(i) $\int \frac{x^2}{(a+bx)^2}dx$ (ii) $\int \frac{x^2+1}{(x+1)^2}dx$

35. Evaluate : $\int \frac{x}{\sqrt{x+a}+\sqrt{x+b}}dx$

36. Evaluate : $\int \cos^2 x\,dx$

37. Evaluate :

(i) $\int \sin^4 x\, dx$ (ii) $\int \cos^4 x\, dx$

38. Evaluate :

(i) $\int \sin 4x \cos 3x\, dx$ (ii) $\int \sin 3x \cos 4x\, dx$

(iii) $\int \cos 2x \cos 4x\, dx$

39. Evaluate :

(i) $\int \cos 2x \cos 4x \cos 6x\, dx$ (ii) $\int \sin x \sin 2x \sin 3x\, dx$

40. Evaluate : $\int \frac{\sin 4x}{\sin x} dx$

41. Integrate the following integrals : $\int \frac{1}{2-3x} + \frac{1}{\sqrt{3x-2}} dx$

42. Integrate the following integrals : $\int \frac{x^3}{x+2} dx$

43. Integrate the following integrals : $\int \sin^2 (2x + 5)\, dx$

44. Integrate the following integrals : $\int \sin^3 (2x + 1)\, dx$

45. Integrate the following integrals : $\int \sin mx \cos nx\, dx\ m \neq n$

46. Integrate the following integrals : $\int \frac{1}{1-\sin\frac{x}{2}} dx$

47. Integrate the following integrals : $\int \frac{1}{\sqrt{x+3}-\sqrt{x+2}} dx$

48. Integrate the following integrals : $\int \frac{2-3x}{\sqrt{1+3x}} dx$

49. Integrate the following integrals : $\int \tan^2 (2x - 3)\, dx$

50. Prove that $\int \text{cosec } x\, dx = \log |\text{cosec } x - \cot x| + C$.

51. Evaluate :

(i) $\int \frac{1}{\sqrt{1+\cos 2x}} dx$ (ii) $\int \sqrt{\frac{1+\cos x}{1-\cos x}} dx$

52. Evaluate :

(i) $\int \tan x \tan 2x \tan 3x\, dx$ (ii) $\int \tan(x - \theta) \tan(x + \theta) \tan 2x\, dx$

53. Evaluate :

(i) $\int \frac{\sin(x-a)}{\sin x}dx$ (ii) $\int \frac{\sin}{\sin(x-a)}dx$

54. Evaluate : $\int \frac{1}{\cos(x-a)\cos(x-b)}dx$

55. Evaluate : $\int \frac{\sin 2x}{\sin\left(x-\frac{\pi}{3}\right)\sin\left(x+\frac{\pi}{3}\right)}dx$

56. Evaluate :

(i) $\int e^{3\log x}(x^4+1)^{-1}\,dx$ (ii) $\int \frac{e^x-e^{-x}}{e^x+e^{-x}}dx$

57. Evaluate : $\int \frac{\sin 2x}{a^2\sin^2 x+b^2\cos^2 x}$

58. Evaluate the following integrals : $\int \sqrt{\frac{1+\cos 2x}{1-\cos 2x}}dx$

59. Evaluate the following integrals : $\int \frac{\sin(x-\alpha)}{\sin(x+\alpha)}dx$

60. Evaluate the following integrals : $\int \frac{e^{3x}}{e^{3x}+1}dx$

61. Evaluate the following integrals : $\int \frac{\sec x\,\mathrm{cosec}\,x}{\log(\tan x)}dx$

62. Evaluate the following integrals : $\int \frac{1-\sin x}{x+\cos x}dx$

63. Evaluate the following integrals : $\int \frac{\cot x}{\log\sin x}dx$

64. Evaluate the following integrals : $\int \frac{e^{2x}}{e^{2x}-2}dx$

65. Evaluate the following integrals : $\int \frac{2\cos x-3\sin x}{6\cos x+4\sin x}dx$

66. Evaluate the following integrals : $\int \frac{\cos 2x}{(\sin x+\cos x)^2}dx$

67. Evaluate the following integrals : $\int \frac{-\sin x + 2\cos x}{2\sin x + \cos x} dx$

68. Evaluate the following integrals : $\int \frac{10x^9 + 10^x \log_e 10}{10^x + x^{10}} dx$

69. Evaluate the following integrals : $\int \frac{e^{x-1} + x^{e-1}}{e^x + x^e} dx$

70. Evaluate the following integrals : $\int \frac{\sec^2 x}{\tan x + 2} dx$

71. Evaluate the following integrals : $\int \frac{2\cos 2x + \sec^2 x}{\sin 2x + \tan x - 5} dx$

72. Evaluate the following integrals : $\int \frac{1 + \tan x}{1 - \tan x} dx$

73. Evaluate the following integrals : $\int \frac{\cos x - \sin x}{1 + \sin 2x}$

74. Evaluate the following integrals : $\int \frac{1}{\sqrt{x}\left(\sqrt{x} + 1\right)} dx$

75. Evaluate the following integrals : $\int \frac{1}{\sin x \cos^2 x} dx$

76. Evaluate the following integrals : $\int \frac{1}{\cos(x + a)\cos(x + b)} dx$

77. Evaluate : $\int \frac{(\log x)^3}{x} dx$

78. Evaluate : $\int \frac{(1 + \log x)^2}{x} dx$

79. Evaluate :

(i) $\int \frac{\tan x \sec^2 x}{(a + b\tan^2 x)^2} dx$ (ii) $\int \sec^3 x \tan x \, dx$

80. Evaluate : $\int \frac{1}{\sqrt{\sin^3 x \sin(x + \alpha)}} dx, \alpha \neq n\pi, n \in Z$

81. Evaluate :

(i) $\int 2^{2^{2^x}} 2^{2^x} 2^x dx$ (ii) $\int \{f(ax+b)\}^n f'(ax+b)dx, n \neq -1$

82. Evaluate : $\int \frac{\sec^4 x}{\sqrt{\tan x}} dx$

83. Evaluate the following integrals : $\int \frac{\left(1+\sqrt{x}\right)^2}{\sqrt{x}} dx$

84. Evaluate the following integrals : $\int \sqrt{1+e^x} e^x dx$

85. Evaluate the following integrals : $\int \cot^3 x \operatorname{cosec}^2 x \, dx$

86. Evaluate the following integrals : $\int \frac{1}{\sqrt{1-x^2}\left(\sin^{-1} x\right)^2} dx$

87. Evaluate the following integrals : $\int \frac{\cos^3 x}{\sqrt{\sin x}} dx$

88. Evaluate the following integrals : $\int \frac{1}{\sqrt{\tan^{-1} x.(1+x^2)}} dx$

89. Evaluate the following integrals : $\int \frac{\sqrt{\tan x}}{\sin x \cos x} dx$

90. Evaluate the following integrals : $\int \frac{1}{2} (\log x)^2 \, dx$

91. Evaluate the following integrals : $\int \frac{1+\cos x}{(x+\sin x)^3 dx}$

92. Evaluate the following integrals : $\int \frac{\cos x - \sin x}{1+\sin 2x} dx$

93. Evaluate the following integrals : $\int \cot x \, . \log \sin x \, dx$

94. Evaluate the following integrals : $\int \operatorname{cosec} x \log (\operatorname{cosec} x - \cot x) \, dx$

95. Evaluate the following integrals : $\int x^2 e^{x^3} \cos(e^{x^3}) \, dx$

96. Evaluate the following integrals : $\int \log x \frac{\sin\left\{1+(\log x)^2\right\}}{x} dx$

97. Evaluate the following integrals : $\int \frac{\sin\sqrt{x}}{\sqrt{x}}dx$

98. Evaluate the following integrals : $\int \frac{\cos\sqrt{x}}{\sqrt{x}}dx$

99. Evaluate the following integrals : $\int \frac{\sin(\tan^{-1}x)}{1+x^2}dx$

100. Evaluate the following integrals : $\int \frac{e^{m\tan^{-1}x}}{1+x^2}dx$

101. Evaluate the following integrals : $\int \frac{1}{x^2(x^4+1)^{\frac{3}{4}}}dx$

102. Evaluate the following integrals : $\int \frac{\cos^8 x}{\sin x}dx$

103. Evaluate the following integrals : $\int \tan^3 2x \sec 2x\, dx$

104. Evaluate : $\int \frac{1}{x^{1/2}+x^{1/3}}dx$

105. Evaluate :

(i) $\int \tan^2 x \sec^4 x\, dx$ (ii) $\int \sec^4 x\, dx$

106. Evaluate : $\int \text{cosec}^4 x\, dx$

107. Evaluate : $\int \tan^3 x \sec^3 x\, dx$

108. Evaluate :

(i) $\int \tan^3 x\, dx$ (ii) $\int \tan^4 x\, dx$

109. Evaluate the following integrals : $\int \tan^3 x \sec^2 x\, dx$

110. Evaluate the following integrals : $\int \tan x \sec^4 x\, dx$

111. Evaluate the following integrals : $\int \tan^5 x\, dx$

112. Evaluate the following integrals : $\int \sqrt{\tan x}\ \sec^4 x\, dx$

113. Evaluate the following integrals : $\int \cot^n \text{cosec}^2 x\, dx$, $n \neq -1$

114. Evaluate the following integrals : $\int \cot^5 x \,\text{cosec}^4 x\, dx$

115. Evaluate the following integrals : $\int \frac{1}{\sin^3 x \cos^5 x}dx$

116. Evaluate the following integrals : $\int \frac{1}{\sin x \cos^3 x} dx$

117. Evaluate :

(i) $\int \frac{1}{4+9x^2} dx$ (ii) $\int \frac{1}{16-9x^2} dx$

118. Evaluate : $\int \frac{1}{\sqrt{9-25x^2}} dx$

119. Evaluate the following integrals : $\int \frac{1}{\sqrt{1+4x^2}} dx$

120. Evaluate the following integrals : $\int \frac{x^4+1}{x^2+1} dx$

121. Evaluate : $\int \frac{1}{2x^2+x-1} dx$

122. Evaluate :

(i) $\int \frac{1}{3x^2+13x-10} dx$ (ii) $\int \frac{1}{x^2+4x+8}$

123. Evaluate the following integrals : $\int \frac{1}{4x^2-4x+3} dx$

124. Evaluate the following integrals : $\int \frac{1}{x^2+6x+13} dx$

125. Evaluate the following integrals : $\int \frac{1}{x^2+x+1} dx$

126. Evaluate the following integrals : $\int \frac{1}{7-6x-x^2} dx$

127. Evaluate the following integrals : $\int \frac{1}{2x^2+7x+13} dx$

128. Evaluate :

(i) $\int \frac{1}{x^4+x^2+1} dx$ (ii) $\int \frac{e^x}{e^{2x}+6e^x+5} dx$

129. Evaluate : $\int \frac{1}{x(x^n+1)} dx$

130. Evaluate the following integrals : $\int \frac{e^{3x}}{4e^{6x}-9}dx$

131. Evaluate the following integrals : $\int \frac{dx}{e^{x}+e^{-x}}$

132. Evaluate the following integrals : $\int \frac{1}{x^4-x^2+1}dx$

133. Evaluate the following integrals : $\int \frac{x}{3x^4-18x^2+11}dx$

134. Evaluate the following integrals : $\int \frac{e^{x}}{(1+e^{x})(2+e^{x})}dx$

135. Evaluate :

(i) $\int \frac{1}{\sqrt{(x-1)(x-2}}dx$ (ii) $\int \frac{1}{\sqrt{9+8x-x^2}}dx$

136. Evaluate : $\int \frac{1}{\sqrt{(x-a)(x-b)}}dx$

137. Evaluate the following integrals : $\int \frac{1}{\sqrt{8+3x-x^2}}dx$

138. Evaluate the following integrals : $\int \frac{1}{\sqrt{5-4x-2x^2}}dx$

139. Evaluate the following integrals : $\int \frac{1}{\sqrt{8+4x-4x^2}}dx$

140. Evaluate the following integrals : $\int \frac{1}{\sqrt{(x+5)(x+1)}}dx$

141. Evaluate the following integrals : $\int \frac{1}{\sqrt{8x-4x^2-3}}dx$

142. Evaluate the following integrals : $\int \frac{1}{\sqrt{(x-\alpha)(\beta-x)}}dx, (\beta > \alpha)$

143. Evaluate the following integrals : $\int \frac{1}{\sqrt{7-3x-2x^2}}dx$

144. Evaluate the following integrals : $\int \frac{1}{\sqrt{16-6x-x^2}}dx$

145. Evaluate the following integrals : $\int \frac{1}{\sqrt{7-6x-x^2}}dx$

146. Evaluate the following integrals : $\int \frac{1}{\sqrt{x^2+12x+11}}dx$

147. Evaluate the following integrals : $\int \frac{1}{\sqrt{5x^2-2x}}dx$

148. Evaluate : $\int \frac{\sec^2 x}{\sqrt{16+\tan^2 x}}dx$

149. Evaluate :

(i) $\int \frac{a^x}{\sqrt{1-a^{2x}}}dx$

(ii) $\int \frac{2x}{\sqrt{1-x^2-x^4}}dx$

(iii) $\int \frac{e^x}{\sqrt{5-4e^x-e^{2x}}}dx$

(iv) $\int \sqrt{\frac{x}{a^3-x^3}}dx$

150. Evaluate :

(i) $\int \sqrt{\sec x-1}\,dx$

(ii) $\int \frac{1}{\sqrt{1-e^{2x}}}dx$

151. Evaluate : $\int \sqrt{\frac{\sin(x-\alpha)}{\sin(x+\alpha)}}dx$

152. Evaluate : $\int \frac{\sec^2 x}{\sqrt{4+\tan^2 x}}dx$

153. Evaluate :

(i) $\int \frac{2\sin 2\phi-\cos\phi}{6-\cos^2\phi=4\sin\phi}$

(ii) $\int \frac{1}{2e^{ex}+3e^x+1}dx$

154. Evaluate the following integrals : $\int \frac{x+2}{2x^2+6x+5}dx$

155. Evaluate the following integrals : $\int \frac{(1-x^2)}{x(1-2x)}dx$

156. Evaluate the following integrals : $\int \frac{x^2+1}{x^2-5x+6}dx$

157. Evaluate the following integrals : $\int \frac{(x-1)^2}{x^2+2x+2}dx$

158. Evaluate the following integrals : $\int \frac{x^2}{x^2+6x+12}dx$

159. Evaluate the following integrals : $\int \frac{x}{\sqrt{x^2+6x+10}}dx$

160. Evaluate the following integrals : $\int \frac{x}{\sqrt{8+x-x^2}}dx$

161. Evaluate the following integrals : $\int \frac{x-1}{\sqrt{x^2+1}}dx$

162. Evaluate the following integrals : $\int \frac{2x+1}{\sqrt{x^2+4x+3}}dx$

163. Evaluate the following integrals : $\int \frac{2x+3}{\sqrt{x^2+4x+5}}dx$

164. Evaluate : $\int \frac{1}{a^2\sin^2 x+b^2\cos^2 x}dx$

165. Evaluate : $\int \frac{1}{3+\sin 2x}dx$

166. Evaluate the following integrals : $\int \frac{1}{4\sin^2 x+5\cos^2 x}dx$

167. Evaluate the following integrals : $\int \frac{1}{(\sin x-2\cos x)(2\sin x+\cos x)}dx$

168. Evaluate the following integrals : $\int \frac{1}{\cos x(\sin x+2\cos x)}dx$

169. Evaluate : $\int \frac{1}{2+\cos x}dx$

170. Evaluate the following integrals : $\int \frac{1}{5+4\cos x}dx$

171. Evaluate the following integrals : $\int \frac{1}{3+2\sin x+\cos x}dx$

172. Evaluate the following integrals : $\int \frac{1}{13+3\cos x+4\sin x}dx$

173. Evaluate the following integrals : $\int \frac{1}{\sqrt{3}\sin x+\cos x}dx$

174. Evaluate the following integrals : $\int \frac{1}{\sin x-\sqrt{3}\cos x}dx$

175. Evaluate the following integrals : $\int \frac{1}{5+7\cos x+\sin x}dx$

176. Evaluate : $\int \frac{1}{1+\cot x}dx$

177. Evaluate : $\int \frac{3\cos x+2}{\sin x+2\cos x+3}dx$

178. Evaluate the following integrals : $\int \frac{1}{1-\tan x}dx$

179. Evaluate the following integrals : $\int \frac{2\tan x+3}{3\tan x+4}$

180. Evaluate the following integrals : $\int \frac{1}{4+3\tan x}dx$

181. Evaluate the following integrals : $\int \frac{8\cot x+1}{3\cot x+2}dx$

182. Evaluate :

(i) $\int x\sec^2 x\,dx$ (ii) $\int x\log x\,dx$

183. Evaluate : $\int \log x\,dx$

184. Evaluate :

(i) $\int \tan^{-1} x\,dx$ (ii) $\int \sec{-1}^{\,x\,dx}$

185. Evaluate :

(i) $\int \frac{\log x}{x^2}dx$ (ii) $\int \frac{x-\sin x}{1-\cos x}dx$

186. Evaluate :

(i) $\int \sec^3 x\,dx$ (ii) $\int (\sin^{-1} x^2)\,dx$

187. Evaluate :

(i) $\int x \log(1+x)\, dx$ (ii) $\int x \sin^{-1} x\, dx$

188. Evaluate : $\int \frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}} dx$

189. Evaluate : $\int \frac{\sqrt{x^2+1}\left[\log(x^2+1) - 2\log x\right]}{x^4} dx$

190. Evaluate : $\int \frac{x^2}{(x\sin x + \cos x)^2} dx$

191. Find an anti–derivative of the function $f(x)\, g''(x) - f''(x)\, g(x)$

192. Evaluate the following integrals : $\int x \cos x\, dx$

193. Evaluate the following integrals : $\int xe^x\, dx$

194. Evaluate the following integrals : $\int x^2 \cos 2x\, dx$

195. Evaluate the following integrals : $\int x \sin 2x\, dx$

196. Evaluate the following integrals : $\int x \sin x \cos x\, dx$

197. Evaluate the following integrals : $\int \sin^{-1}(3x - 4x^3)\, dx$

198. Evaluate the following integrals : $\int \sin^{-1}\left(\frac{2x}{1+x^2}\right) dx$

199. Evaluate the following integrals : $\int \tan^{-1}\left(\frac{3x - x^3}{1-3x^2}\right) dx$

200. Evaluate the following integrals : $\int \cos^{-1}(4x^3 - 3x)\, dx$

201. Evaluate the following integrals : $\int \tan^{-1}\left(\frac{2x}{1-x^2}\right) dx$

202. Evaluate the following integrals : $\int \tan^{-1}\sqrt{\frac{1-x}{1+x}}\, dx$

203. Evaluate the following integrals : $\int \sin^{-1}\sqrt{\frac{x}{a+x}}\, dx$

204. Evaluate the following integrals : $\int \frac{x^3 \sin^{-1} x^2}{\sqrt{1-x^4}} dx$

205. Evaluate the following integrals : $\int (x+1) \log x\, dx$

206. Evaluate the following integrals : $\int \frac{\sin^{-1} x}{x^2} dx$

207. Evaluate :

(i) $\int e^x \left(\frac{1}{x} - \frac{1}{x^2}\right) dx$

(ii) $\int e^x \frac{x}{(x+1)^2} dx$

208. Evaluate :

(i) $\int e^x (\sin x + \cos x)\, dx$

(ii) $\int e^x \left(\frac{2+\sin 2x}{1+\cos 2x}\right) dx$

(iii) $\int e^x \left(\frac{1-\sin x}{1-\cos x}\right) dx$

209. Evaluate : $\int \{\sin (\log x) + \cos (\log x)\}\, dx$

210. Evaluate : $\int e^{2x} \left(\frac{1+\sin 2x}{1+\cos 2x}\right) dx$

211. Evaluate the following integrals : $\int e^{2x} \left(\frac{1+\sin x}{1+\cos x}\right) dx$

212. Evaluate the following integrals : $\int e^x (\cot x - \operatorname{cosec}^2 x)\, dx$

213. Evaluate the following integrals : $\int e^x \left(\frac{\sin 4x - 4}{1-\cos 4x}\right) dx$

214. Evaluate the following integrals : $\int e^x \left(\frac{(1-x)^2}{(1+x^2)^2}\right) dx$

215. Evaluate the following integrals : $\int e^x \left(\log x + \frac{1}{x^2}\right) dx$

216. Evaluate the following integrals : $\int \left(\frac{1}{\log x} - \frac{1}{(\log x)^2}\right)$

217. Evaluate the following integrals : $\int e^x \left(\frac{\sin x \cos x - 1}{\sin^2 x}\right) dx$

218. Evaluate the following integrals : $\int \{\tan (\log x) + \sec^2 (\log x)\}\, dx$

219. Evaluate the following integrals : $\int e^{2x} \left(\frac{x-3}{(x-1)^3}\right) dx$

220. Prove that : $\int e^{ax} \sin bx \, dx = \frac{e^{ax}}{a^2+b^2}(a \sin bx - b \cos bx) + c$

221. Evaluate : $\int e^{2x} \sin 3x \, dx$

222. Evaluate : $\int e^{ax} \cos (bx + c) \, dx$

223. Evaluate the following integrals : $\int e^{ax} \cos bx \, dx$

224. Evaluate the following integrals : $\int e^{2x} \sin x \cos x \, dx$

225. Evaluate the following integrals : $\int e^{2x} \sin x \, dx$

226. Evaluate the following integrals : $\int e^{x} \sin^2 x \, dx$

227. Evaluate the following integrals : $\int e^{2x} \cos^2 x \, dx$

228. Prove that :

(i) $\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}x\sqrt{a^2-x^2} + \frac{1}{2}a^2 \sin^{-1}\left(\frac{x}{a}\right) + C$

(ii) $\int \sqrt{a^2+x^2}\,dx = \frac{1}{2}x\sqrt{a^2+x^2} + \frac{1}{2}a^2 \log\left|x+\sqrt{a^2+x^2}\right| + C$

(iii) $\sqrt{x^2-a^2}\,dx = \frac{1}{2}x\sqrt{x^2-a^2} + \frac{1}{2}a^2 \log\left|x+\sqrt{x^2-a^2}\right| + C$

229. Evaluate the following integrals : $\int \sqrt{3+2x-x^2}\,dx$

230. Evaluate the following integrals : $\int \sqrt{1+x-2x^2}\,dx$

231. Evaluate the following integrals : $\int e^{x}\sqrt{e^{2x}+1}\,dx$

232. Evaluate the following integrals : $\int \sqrt{16x^2+25}\,dx$

233. Evaluate the following integrals : $\int \sqrt{3-2x-2x^2}\,dx$

234. Evaluate the following integrals : $\int \frac{\sqrt{16+(\log x)^2}}{x}\,dx$

235. Evaluate the following integrals : $\int \sqrt{2ax-x^2}\,dx$

236. Evaluate the following integrals : $\int \sqrt{3-x^2}\,dx$

237. Evaluate : $\int (x+1)\sqrt{1-x-x^2}\,dx$

238. Evaluate the following integrals : $\int (x+1)\sqrt{x^2-x+1}\,dx$

239. Evaluate the following integrals : $\int (2x-5)\sqrt{x^2-4x+3}\,dx$

240. Evaluate : $\int \frac{x-1}{(x+1)(x-2)}dt$

241. Evaluate : $\int \frac{2x}{(x^2+1)(x^2+2)}dx$

242. Evaluate : $\int \frac{\tan\theta+\tan^3\theta}{1+\tan^3\theta}d\theta$

243. Evaluate : $\int \frac{\sin x}{\sin 4x}dx$

244. Evaluate the following integrals : $\int \frac{3+4x-x^2}{(x+2)(x-1)}dx$

245. Evaluate the following integrals : $\int \frac{5x}{(x+1)(x^2-4)}dx$

246. Evaluate the following integrals : $\int \frac{2x}{(x^2+1)(x^2+3)}dx$

247. Evaluate the following integrals : $\int \frac{1}{\cos x(5-4\sin x)}dx$

248. Evaluate the following integrals : $\int \frac{1}{(x-1)(x+1)(x+2)}dx$

249. Evaluate the following integrals : $\int \frac{x^2+1}{(2x+1)(x^2-1)}dx$

250. Evaluate the following integrals : $\int \frac{1}{x([6(\log x)^2+7\log x+2]}dx$

251. Evaluate the following integrals : $\int \frac{1}{x(x^n+1)}dx$

252. Evaluate the following integrals : $\int \frac{x}{(x-1)^2(x+2)}dx$

253. Evaluate the following integrals : $\int \frac{x^2}{(x-1)(x+1)^2}dx$

254. Evaluate the following integrals : $\int \frac{x}{(x+1)(x^2+1)}dx$

255. Evaluate the following integrals : $\int \frac{x^3-1}{x^3+x} = dx$

256. Evaluate the following integrals : $\int \frac{2x+1}{(x-2)(x-3)} dx$

257. Evaluate the following integrals : $\int \frac{1}{(x^2+1)(x^2+2)} dx$

258. Evaluate the following integrals : $\int \frac{3x+5}{x^3-x^2-x+1} dx$

259. Evaluate :

(i) $\int \frac{x^2+1}{x^4+1} dx$

(ii) $\int \frac{x^2+4}{x^4+16} dx$

260. Evaluate : $\int \sqrt{\tan\theta}\, d\theta$

261. Evaluate : $\int \left\{\sqrt{\tan\theta} + \sqrt{\cot\theta}\right\} d\theta$

262. Evaluate : $\int \frac{1}{\sin^4 x + \cos^4 x} dx$

263. Evaluate : $\int \frac{1}{(x^2-4)\sqrt{x+1}} dx$

264. Evaluate the following integrals : $\int \frac{1}{(x-1)\sqrt{x+2}} dx$

265. Evaluate the following integrals : $\int \frac{x^2}{(x-1)\sqrt{x+2}} dx$

266. Evaluate the following integrals : $\int \frac{1}{(2x^2+3)\sqrt{x^2-4}} dx$

267. Evaluate the following integrals : $\int \frac{1}{(x^2+4)\sqrt{x^2+9}} dx$

EXERCISE-2

Answer each of the following questions in one word or one sentence or as per exact requirement of the question :

1. Write a value of $\int x^2 \sin x^3\, dx$

2. Write a value of $\int \tan^3 x \sec^2 x\, dx$

3. Write a value of $\int \frac{(\log x)^n}{x} dx$

4. Write a value of $\int \tan x \sec^3 x \, dx$

5. Write a value of $\int \frac{1}{1+e^x} dx$

6. Write a value of $\int \frac{(\tan^{-1} x)^3}{1+x^2} dx$

7. Write a value of $\int \frac{\sec^2 x}{(5+\tan x)^4} dx$

8. Write a value of $\int \frac{\cos x}{\sin x \log \sin x} dx$

9. Write a value of $\int \frac{1}{x(\log x)^n} dx$

10. Write a value of $\int e^{ax} \{af(x) + f'(x)\} dx$

11. Evaluate : $\int \frac{\sec^2 \sqrt{x}}{\sqrt{x}} dx$

12. Evaluate : $\int \frac{\sin \sqrt{x}}{\sqrt{x}} dx$

13. Evaluate : $\int \frac{(1+\log x)^2}{x} dx$

14. Evaluate : $\int \frac{1-\sin x}{\cos^2 x} dx$

15. Evaluate the following integrals : $\int \frac{1-x^4}{1-x} dx$

16. Evaluate the following integrals : $\int \frac{8x-13}{\sqrt{4x+7}} dx$

17. Evaluate the following integrals : $\int \sec^2 x \cos^2 2x \, dx$

18. Evaluate the following integrals : $\int \frac{(\sin^{-1} x)^3}{\sqrt{1-x^2}} dx$

19. Evaluate the following integrals : $\int \frac{\cos^7 x}{\sin x} dx$

20. Evaluate the following integrals : $\int \frac{\sin x + \cos x}{\sqrt{\sin 2x}} dx$

21. Evaluate the following integrals : $\int \frac{1}{\cos(x-a)\cos(x-b)} dx$

22. Evaluate the following integrals : $\int \tan^4 x \, dx$

23. Evaluate the following integrals : $\int \cot^4 x \, dx$

24. Evaluate the following integrals : $\int \frac{\sin 2x}{\sin^4 x + \cos^4 x} dx$

25. Evaluate the following integrals : $\int \frac{1}{\sqrt{x^2 + a^2}} dx$

26. Evaluate the following integrals : $\int \frac{5x+7}{\sqrt{(x-5)(x-4)}} dx$

27. Evaluate the following integrals : $\int \sqrt{\frac{1+x}{x}} dx$

28. Evaluate the following integrals : $\int \sqrt{\frac{1-x}{x}} dx$

29. Evaluate the following integrals : $\int \frac{1}{a + b\tan x} dx$

30. Evaluate the following integrals : $\int \frac{1}{2 - 3\cos 2x} dx$

31. Evaluate the following integrals : $\int \frac{1}{1 + 2\cos x} dx$

32. Evaluate the following integrals : $\int \frac{1}{2 + \cos x} dx$

33. Evaluate the following integrals : $\int \frac{6x+5}{\sqrt{6 + x - 2x^2}} dx$

34. Evaluate the following integrals : $\int \frac{\cos^5 x}{\sin x} dx$

35. Evaluate the following integrals : $\int \sec^6 x \, dx$

36. Evaluate the following integrals : $\int \tan^3 x \sec^4 x \, dx$

37. Evaluate the following integrals : $\int x \sin^3 x \, dx$

38. Evaluate the following integrals : $\int \frac{\log x}{x^3} dx$

39. Evaluate the following integrals : $\int \frac{\log(1-x)}{x^2} dx$

40. Evaluate the following integrals : $\int x^3 (\log x)^2 \, dx$

41. Evaluate the following integrals : $\int x\sqrt{\frac{1-x}{1+x}} dx$

42. Evaluate the following integrals : $\int \frac{\sin x + \cos x}{\sin^4 x + \cos^4 x} dx$

43. Evaluate the following integrals : $\int \sec^{-1} \sqrt{x} \, dx$

44. Evaluate the following integrals : $\int \tan^{-1} \sqrt{\frac{1-x}{1+x}} dx$

45. Evaluate the following integrals : $\int \frac{\sqrt{1-\sin x}}{1+\cos x} e^{-x/2} dx$

46. Evaluate the following integrals : $\int e^x \frac{(1-x)^2}{(1+x^2)^2} dx$

47. Evaluate the following integrals : $\int \frac{1}{(x^2+2)(x^2+5)} dx$

48. Evaluate the following integrals : $\int \frac{x^2-2}{x^5-x} dx$

49. Evaluate the following integrals : $\int \frac{\sin x + \cos x}{9+16\sin 2x} dx$

50. Evaluate the following integrals : $\int \sqrt{\frac{\sin(x-\alpha)}{\sin(x+\alpha)}} dx$

51. Evaluate the following integrals : $\int \sqrt{e^x - 1} \, dx$

52. Evaluate the following integrals : $\int \frac{\cot x + \cot^3 x}{1+\cot^3 x} dx$